Hybrid RRT: Motion Planning for Hybrid Dynamical Systems

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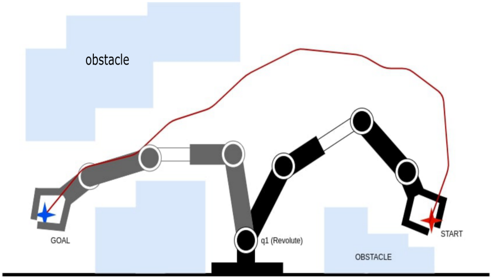
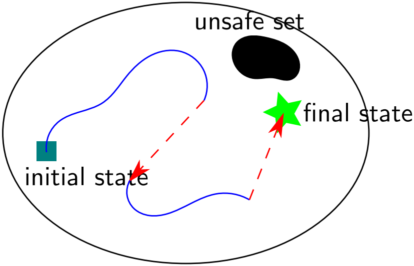
1. Motivation: robotics motion planning



1. Motivation: robots with hybrid dynamics

The motion planning problem for hybrid systems is to find a trajectory of states and inputs that starts from the initial state (set), ends within the final state (set), and satisfies both continuous and discrete dynamics and **safety** criterion.



Outline

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2. **Preliminaries on Hybrid System Modeling**
3. **Problem Formulation**
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   - Probabilistic completeness
   - Simulation results

2. Preliminaries on Hybrid System Modeling

A hybrid system $\mathcal{H}$ with inputs is modeled by a system of differential and difference equations as

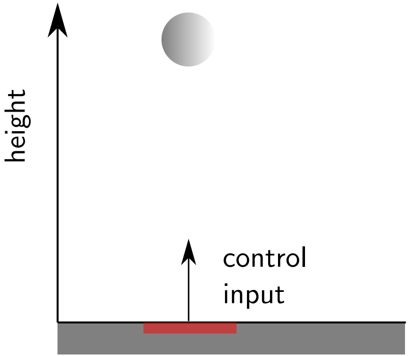
$$\mathcal{H} : \begin{cases} \dot{x} = f(x,u) & (x,u) \in C \\ x^+ = g(x,u) & (x,u) \in D \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is state, $u \in \mathbb{R}^m$ is input,

- $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the flow map;
- $C \subset \mathbb{R}^n \times \mathbb{R}^m$ is the flow set;
- $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the jump map;
- $D \subset \mathbb{R}^n \times \mathbb{R}^m$ is the jump set.

2. Preliminaries on Hybrid System Modeling

A motivating example: A classic example of hybrid system is bouncing ball system. Consider a ball bouncing on a fixed horizontal surface. The surface is capable of affecting the velocity of the ball after the impact through control actions.



2. Preliminaries on Hybrid System Modeling

A motivating example: A classic example of hybrid system is bouncing ball system. Consider a ball bouncing on a fixed horizontal surface. The surface is capable of affecting the velocity of the ball after the impact through control actions.

- x_1 = height, x_2 = velocity
- γ = the gravity constant, λ = coefficient of restitution

$$\mathcal{H} : \begin{cases} \dot{x} = \begin{bmatrix} x_2 \\ -\gamma \end{bmatrix} =: f(x, u) & \forall (x, u) \in C \\ x^+ = \begin{bmatrix} x_1 \\ -\lambda x_2 + u \end{bmatrix} =: g(x, u) & \forall (x, u) \in D \end{cases} \tag{2}$$

where

$$C := \{(x, u) \in \mathbb{R}^2 \times \mathbb{R} : x_1 \geq 0\}$$
$$D := \{(x, u) \in \mathbb{R}^2 \times \mathbb{R} : x_1 = 0, x_2 \leq 0, u \geq 0\}.$$

2. Preliminaries on Hybrid System Modeling

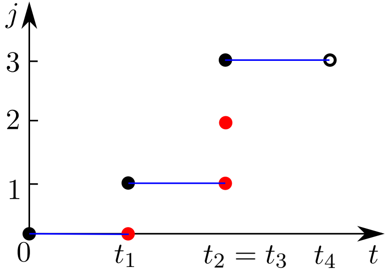
Hybrid time domain: The solutions and inputs to a hybrid system $\mathcal{H}$ are parameterized by (t, j) where

- $t \in \mathbb{R}_{\geq 0}$ denotes the normal time variable
- $j \in \mathbb{N}$ denotes the number of jumps.

Hybrid time domains:

$$\operatorname{dom}\psi := ([0, t_1]) \times \{0\} \cup ([t_1, t_2]) \times \{1\} \cup \cdots \cup ([t_j, t_{j+1}]) \times \{j\} \cup \cdots$$

for some finite sequence of times $0 = t_0 \leq t_1 \leq t_2 \leq ... \leq t_{J+1}$.



2. Preliminaries on Hybrid System Modeling

Definition 2.1 (Solution pair to a hybrid system (**altin2019hybrid**))

Given a pair of functions $\phi : \operatorname{dom}\phi \to \mathbb{R}^n$ *and* $u : \operatorname{dom} u \to \mathbb{R}^m$ *defined on hybrid time domains,* (ϕ, u) *is a solution pair to hybrid system* $\mathcal{H} = (C, f, D, g)$ *if:*

1. *During flows,*

$$(\phi(t,j), u(t,j)) \in C \quad \dot{\phi}(t,j) = f(\phi(t,j), u(t,j))$$

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2. *At jumps,*

$$(\phi(t,j), u(t,j)) \in D \quad \phi(t,j+1) = g(\phi(t,j), u(t,j)).$$

3. Problem Formulation

Problem 1 (Motion planning problem for hybrid systems)

Given

1. *a hybrid system* $\mathcal{H} = (C, f, D, g)$ *with input* $u \in \mathbb{R}^m$, *state* $x \in \mathbb{R}^n$;

find a pair $(\phi, u) : \mathrm{dom}(\phi, u) \to \mathbb{R}^n \times \mathbb{R}^m$, *namely, a motion plan, such that:*

1. (ϕ, u) *is a solution pair to* $\mathcal{H}$;

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1. *a hybrid system* $\mathcal{H} = (C, f, D, g)$ *with input* $u \in \mathbb{R}^m$, *state* $x \in \mathbb{R}^n$;
2. *initial state set* $X_0 \subset \mathbb{R}^n$;

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1. (ϕ, u) *is a solution pair to* $\mathcal{H}$;
2. $\phi(0, 0) \in X_0$;
3. *there exists* $(T, J) \in \mathrm{dom}(\phi, u)$ *such that* $\phi(T, J) \in X_f$;

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Given

1. *a hybrid system* $\mathcal{H} = (C, f, D, g)$ *with input* $u \in \mathbb{R}^m$, *state* $x \in \mathbb{R}^n$;
2. *initial state set* $X_0 \subset \mathbb{R}^n$;
3. *final state set* $X_f \subset \mathbb{R}^n$;
4. *unsafe set* $\mathbf{X_u} \subset \mathbb{R}^n \times \mathbb{R}^m$;

find a pair $(\phi, u) : \mathrm{dom}(\phi, u) \to \mathbb{R}^n \times \mathbb{R}^m$, *namely, a motion plan, such that:*

1. (ϕ, u) *is a solution pair to* $\mathcal{H}$;
2. $\phi(0, 0) \in X_0$;
3. *there exists* $(T, J) \in \mathrm{dom}(\phi, u)$ *such that* $\phi(T, J) \in X_f$;
4. $(\phi(t, j), u(t, j)) \notin \mathbf{X_u}$ *for each* $(t, j) \in \mathrm{dom}(\phi, u)$, $t + j \leq T + J$.

4. HyRRT Algorithm

4.1 Algorithm description

Algorithm 1 HyRRT algorithm

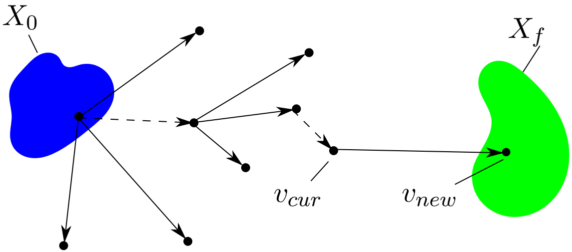
Input: $X_0, X_f, X_u, \mathcal{H} = (C, f, D, g), \mathcal{U} = (\mathcal{U}_C, \mathcal{U}_D), p_n \in (0, 1)$

$\mathcal{T}.init(X_0)$
for $i = 1$ to k **do**
    randomly select a real number n from $[0, 1]$
    if $n \leq p_n$ **then**
        $x_{rand} \leftarrow random\_state(\overline{C'})$
        $extended \leftarrow extend(\mathcal{T}, x_{rand}, \mathcal{U}, \mathcal{H}, X_u, flow)$
    else
        $x_{rand} \leftarrow random\_state(D')$
        $extended \leftarrow extend(\mathcal{T}, x_{rand}, \mathcal{U}, \mathcal{H}, X_u, jump)$
    end if
    if $extended == 1 \& check\_solution(\mathcal{T}, X_0, X_f, C, \psi_{sol}) == 1$ **then**
        return ψ_{sol}
    end if
end for

4. HyRRT Algorithm

4.1 Algorithm description

The propagation results:



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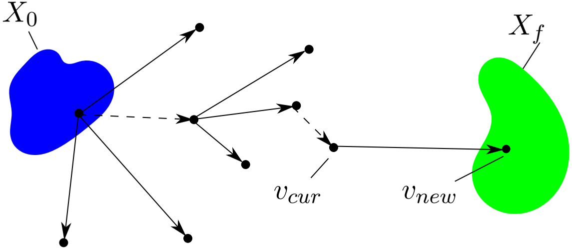

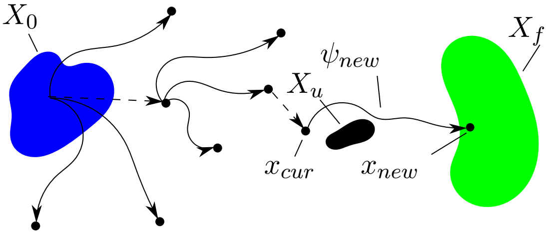

4. HyRRT Algorithm

4.2 Probabilistic completeness

Definition 4.1

*(Probabilistic completeness (***kleinbort2018probabilistic***)) A sampling-based algorithm is said to be probabilistically complete if the probability of failing to find a solution is converging to* 0*, as the number of samples approaches to infinity.*

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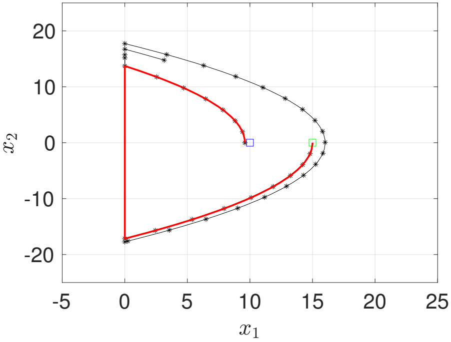
Theorem 1

The proposed HyRRT is probabilistically complete for any given motion planning problem for hybrid systems formulated as in Problem 1.

4. HyRRT Algorithm

4.3 Simulation results: Algorithm 1 leads to a HyRRT software tool[1] to solve the motion planning problems for hybrid systems. The simulation is implemented in MATLAB software and processed by a 2.2 GHz Intel Core i7 processor. The simulation takes 0.34 seconds to finish.



[1] Code at https://github.com/HybridSystemsLab/hybridRRT.

4. HyRRT Algorithm

4.3 Simulation results: The simulation is implemented in MATLAB software and processed by a 3.5 GHz Intel Core i5 processor. The simulation takes 57.6 seconds to finish.

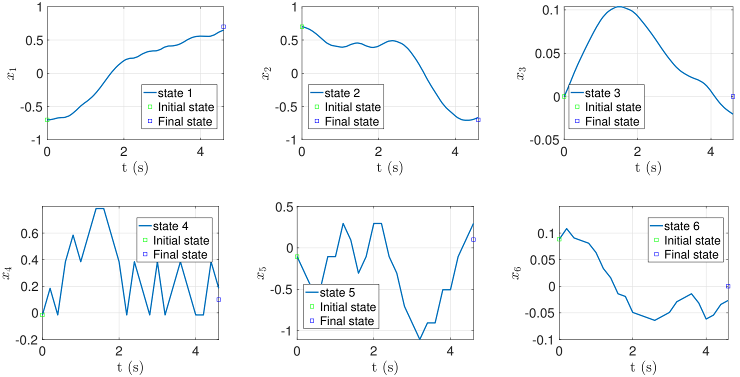


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Table: Computation Costs of HyRRT and FBP in the Biped Example .

	Time Consumption (seconds)	Vertices
HyRRT	57.6	2357
FBP	1608.2	3796

Acknowledgement

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Thank you for your attention. Any questions?